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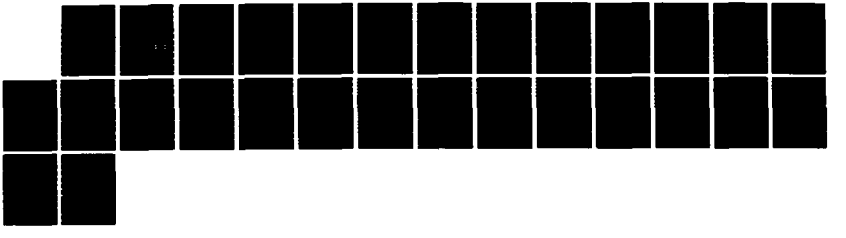

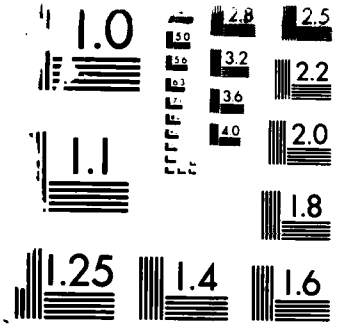

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SYNTHESIS OF PARTITIONABLE MULTISTAGE NETWORKS FINAL REPORT

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Abstract

Two methods have been used to speed up the execution of computation. One is the technology insertion and the other is new architectural concepts. Regardless of the type of architecture development the result is a parallel computer systems with a large number of processing elements. The communication requirements between the processing elements will lead to the need for a large interconnection networks.

In this research a property of interconnection networks called partitionability is studied. The advantages and uses of partitionable networks were described in number of papers. The partitionability informally means that the system can be divided into several parts each of which has certain amount of behavioral independence.

Several researchers have analyzed both topologically regular and irregular interconnection networks with respect to the partitionability property.

In this work the concern is synthesis techniques of partitionable networks. Two algorithms are developed each capable of synthesizing a class of partitionable interconnection networks. The generated classes are informally described.

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I Introduction

Two methods have been used to speed up the execution of computation. One is the technology insertion and the other is new architectural concepts. Regardless of the type of architecture development the result is a parallel computer systems with a large number of processing elements. The communication requirements between the processing elements will lead to the need for a large interconnection networks.

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In this research a property of interconnection networks called partitionability is studied. The partitionability informally means that the system can be divided into several parts each of which has certain amount of behavioral independence.

The partitionability of Banyan networks was shown in [Gok76, GoL73]. The partitionability of the Cube network was analyzed in [SiM81b] and ADM in [SiM81a]. Partitionability of topologically arbitrary single stage networks was studied in [SeS85]. The quotient property (which is related to partitionability) of several well known networks was studied in [FiF82].

As it can be seen, most work has been done in analysis and little has been done in the synthesis area. In this paper the synthesis of multistage partitionable interconnection networks is studied. Two algorithms are developed each of which is capable of generating a class of partitionable networks.

Partitionable networks have the following uses and advantages. They can be used to allocate a proportional number of processors to the computational and communication needs of a task in a multitasking system. This method of allocation is used in systems such as PASM currently developed at Purdue University [SiS81]. The partitioning property provides a natural protection amongst users in a multiuser environment. That is accomplished by giving each user a segment of the system which will provide a hardware protection amongst users. The implementation of the routing of the data path and control is efficient on VLSI substrate or on printed circuit board (PCB). In the case of fault in a part of the network, a method of graceful degradation is possible by an easy migration of the program to the correctly operating sections. The partitionable network forms the basis of a certain class of fault tolerant networks. The network Extra Stage Cube [AdS82] has a type of fault tolerance that can be traced to the necessary condition of partitionability of the core of the network. The Extra Stage Cube is single switch, single link fault tolerant however the idea can be generalized into

multiple faults using a properly partitionable network core. In the Star system the baseline partitionable network is used because of its ability to establish partitions corresponding to the computational structures of the algorithm [AgY85, WuF82]. The partitionable property of networks is an essential element in the modular expansion of dynamic systems [ViK80].

II Overview

In Section III the basic notation and definitions are presented. In Section IV the necessary background on single stage network is presented and partitionability properties of single stage networks are described. In Section V the multistage networks are discussed. A graph/algebraic model is developed and two classes of partitionable networks are defined. A methodology to synthesize each class is presented and proven correct. In Section VI the global conclusion is presented.

III Basic Concepts

In this section, basic definitions and notation needed as the background for the rest of the paper are introduced. Some of the definitions can be found in books on basic abstract algebra [Han68, Her75] and graph theory [BoM76, Har69], however are included here for completeness. This material was developed in [SeS84].

Let the set of input labels of a graph/algebraic structure be denoted by V_I and the set of output labels of the structure be denoted by V_O. All graph/algebraic structures defined in this paper over $V_I \times V_O$ will assume that $V_I \cap V_O = \emptyset$, $V_I \neq \emptyset$, $V_O \neq \emptyset$,

where Ø is the *empty set* and $V_I \times V_O = \{<v_a,v_b> | v_a \in V_I, v_b \in V_O\}$.

The following notation will be used throughout this paper. The symbols are enclosed in a pair of double quotation marks.

"{","}" - delimiters for set. "(",")" - function application and grouping of operations. "<",">" - delimiters for n-tuple. "[","]" - used as defined in context.

Definition 3.1:

Let A be a set, then $P[A] \triangleq \{B \mid B \subseteq A\}$ is the *power set of A*.

Definition 3.2:

Let $C_m \in P[V_I \times V_O]$, then C_m is an *I/O correspondence over* $V_I \times V_O$.

Definition 3.3:

Let $C_m \in P[V_I \times V_O]$ such that $<v_a,v_b>, <v_c,v_d> \in C_m \Rightarrow v_b \neq v_d$, then the C_m is a *nondestructive I/O correspondence over* $V_I \times V_O$. (Physically, C_m represents one state of a network).

Definition 3.4:

Let $C[V_I \times V_O] \triangleq \{C_m \in P[V_I \times V_O] \mid C_m \text{ is nondestructive}\}$. Then $C[V_I \times V_O]$ is called the *C-set over* $V_I \times V_O$.

Definition 3.5:

Let $C_m \in C[V_I \times V_O]$, then $s(C_m) \triangleq \{v_a | <v_a,v_b> \in C_m\}$ is the *source set of* C_m.

Definition 3.6:

Let $C_m \in C[V_I \times V_O]$, then $d(C_m) \triangleq \{v_b | <v_a,v_b> \in C_m\}$ is the *destination set of* C_m.

Definition 3.7:

Let $C = \{C_m \mid m=1,2,...n\} \subseteq C[V_I \times V_O]$, then $s(C) \triangleq \bigcup_m s(C_m)$ is the *source set of C* and $d(C) \triangleq \bigcup_m d(C_m)$ is the *destination set of C.*

Definition 3.8:

Let V be a set of labels. Let $E \subseteq V \times V$, then $G = <V,E>$ is called a *graph.*

IV Single Stage Interconnection Networks

This section consists of three subsections. The first subsection describes the model of single stage network used later to construct multistage networks. The second subsection defines and discusses the basic composition and decomposition of networks. The third subsection uses the decomposition concepts to define partitionability of single stage networks which is used in the next section to discuss the partitionability of multistage networks. Partitionability informally means that the network can be decomposed into two parts with certain degree of independence. The definitions and theorems discussed in this section were developed in [SeS84, SeS85] and are presented here (without proofs) for completeness.

In this subsection, a formal graph/algebraic model of an interconnection network is presented. Graph models for analyzing networks have been used by other researchers. For example, in [Gok76, GoL73, LiM82, Upp81] they are used to analyze regular SW Banyan networks, and in [FiF82] they are used to study the partitioning of regular networks. The model presented here differs from [Gok76, GoL73, LiM82, Upp81] and [FiF82] by being completely general so that it can be used to describe an arbitrary, topologically regular and irregular, interconnection network. The model is similar to

the one used by [MaM81] to study time space tradeoffs.

Definition 4.1:

Let $K = <C>$ be such that:

(1) $C \subseteq C[V_I \times V_O]$.

(2) $V_I = s(C)$.

(3) $V_O = d(C)$.

If $|C| \geq 2$ then $K = <C>$ is an *I/O representation of a reconfigurable network over* $V_I \times V_O$. If $|C| = 1$ then $K = <C>$ is an *I/O representation of a fixed network over* $V_I \times V_O$.

Physical implications: $<v_a,v_b> \in C_m$, $C_m \in C$ represents the network moving data from input v_a to output v_b when the state of the network is C_m. C represents the set of all possible states of the reconfigurable network. For an example of a topologically arbitrary interconnection network see Figure 1. The example has the following parameters:

$V_I = \{u_a, u_b, u_c\}$, $V_O = \{v_0, v_1\}$, $C_0 = \{<u_a,v_0> <u_a,v_1>\}$.
$C_1 = \{<u_a,v_0>, <u_b,v_1>\}$, $C_2 = \{<u_a,v_1>, <u_c,v_0>\}$,
$C = \{C_0, C_1, C_2\}$. $K = <\{C_0, C_1, C_2\}>$.

Definition 4.2:

Let $K[V_I \times V_O] \triangleq \{K \mid K = <C>$ is a network over $V_I \times V_O\}$. Then $K[V_I \times V_O]$ is called the *K-set over* $V_I \times V_O$.

Definition 4.3:

Let $K^1 \in K[V_I^1 \times V_O^1]$, $K^1 = <C^1>$, and $K^2 \in K[V_I^2 \times V_O^2]$, $K^2 = <C^2>$, be two networks such that:

(1) $V_I^1 \subseteq V_I^2, V_O^1 \subseteq V_O^2$.

(2) $\forall C_m^1 \in C^1 \quad \exists C_n^2 \in C^2 \ni: C_m^1 = C_n^2$.

Then K^1 *is subnetwork of type c of* K^2. *Notation:* $K^1 \subseteq c\ K^2$. The other types are not used here and therefore they are not presented.

This subsection describes an "intra-stage" composition and decomposition of single stage networks. The discussion here is presented for the composition of two networks into one and the decomposition of one network into two. However, it can be generalized into the composition of n networks into one and decomposition of one network into n, $n > 2$. What is meant by the *intra-stage composition* of two networks K^1 and K^2 is that $V_I^1 \bigcap V_I^2 = \emptyset$ and $V_O^1 \bigcap V_O^2 = \emptyset$. Similarly, the *intra-stage decomposition* of K into two networks K^1 and K^2 will result in $V_I^1 \bigcap V_I^2 = \emptyset$ and $V_O^1 \bigcap V_O^2 = \emptyset$. Two types of composition (decomposition) are described. One, the *σ-composition (decomposition)* corresponds to the physical situation where the controls of the individual subnetworks of the network are independent. The other type is the *τ-composition (decomposition),* which corresponds to the physical situation where the controls of the individual subnetworks of the network are dependent upon one another.

Definition 4.4:

Let $K^1 \in K[V_I^1 \times V_O^1]$, $K^1 = <C^1>$, and $K^2 \in K[V_I^2 \times V_O^2]$, $K^2 = <C^2>$, be two networks such that: $(V_I^1 \bigcup V_O^1) \bigcap (V_I^2 \bigcup V_O^2) = \emptyset$. Define *$\sigma$-map* as follows: $K^1 \sigma K^2 = <C^1> \sigma <C^2> \triangleq <\{C_p^1 \bigcup C_r^2 \mid C_p^1 \in C^1, C_r^2 \in C^2\}>$.

This describes the composition of two networks where the controls of the two networks are independent from one another.

Definition 4.5:

Let $K \in K[V_I \times V_O]$ be a network. Let $\{K^1, K^2, ..., K^n \mid K^i \in K[V_I^i \times V_O^i]\}$ be a set of networks such that: $K \subseteq_c K^1 \sigma K^2 \sigma \cdots K^n$.

Notice that this implies $V_I = \dot{\bigcup_{i=1}^{n}} V_I^i$ and $V_O = \dot{\bigcup_{i=1}^{n}} V_O^i$. Then

(1) $K^1 \sigma K^2 \sigma \cdots K^n$ is called a *σ-decomposition of K.*

(3) K^i is called a *component network of K* (K^i is maybe itself decomposable).

In this second part of this subsection, the τ-composition and decomposition of two networks will be discussed. In the σ-composition, the two networks keep independent controls, that is if C_m^1 is selected in K^1, an arbitrary correspondence C_n^2 can be selected in K^2. In the τ-composition, the two networks have joint control, that is if C_m^1 is selected in K^1, the corresponding C_n^2 must be selected in K^2.

Definition 4.6:

Let $K^1 \in K[V_I^1 \times V_O^1]$, $K^1 = <C^1>$, and $K^2 \in K[V_I^2 \times V_O^2]$, $K^2 = <C^2>$ be two networks such that:

(a) $(V_I^1 \cup V_O^1) \cap (V_I^2 \cup V_O^2) = \emptyset$, and (b) $|C^1| = |C^2|$.

Define *τ_α-map* as follows:

(1) Define α: $C^1 \rightarrow C^2$, map 1:1 and onto.

(2) $K^1 \tau_\alpha K^2 = <C^1> \tau_\alpha <C^2> \triangleq <\{C_p^1 \cup C_r^2 \mid \alpha(C_p^1) = C_r^2, \; C_p^1 \in C^1, \; C_r^2 \in C^2\}>$.

Definition 4.7:

Let $K \in K[V_I \times V_O]$ be a network. Let $\{K^1, K^2, ..., K^n \mid K^i \in K[V_I^i \times V_O^i]\}$ be a set of networks such that: $K = K^1 \tau_\alpha K^2 \tau_\alpha \cdots K^n$. Then (1) $K^1 \tau_\alpha K^2 \tau_\alpha \cdots K^n$ is called a *τ-decomposition of K.*

(3) K^i is called a *component network of K.* (K^i maybe itself decomposable).

Definition 4.8:

Let $K \in K[V_I \times V_O]$, $K = <C>$ be a network. K is a *prime* network iff K cannot be decomposed as $K \subseteq_c K^1 \sigma K^2$.

Definition 4.9:

Let $K \in K[V_I \times V_O]$, $K = <C>$ be a network. If there exist $K^1 \in K[V_I^1 \times V_O^1]$, $K^1 = <C^1>$, and $K^2 \in K[V_I^2 \times V_O^2]$, $K^2 = <C^2>$, two prime networks such that: (1) $V_I^1 \cup V_I^2 = V_I$, and (2) $V_O^1 \cup V_O^2 = V_O$, then:

(1) If $K^1 \tau_a K^2 = K$, then K is a *τ-partitionable* network.

(2) If $K^1 \sigma K^2 = K$, then K is a *strictly σ-partitionable* network.

(3) If $K^1 \sigma K^2 \supseteq_c K$, then K is a *σ-partitionable* network.

Note that τ-partitionable implies $|C| = |C^1| = |C^2|$, strictly σ-partitionable implies $|C| = |C^1| \times |C^2|$, and σ-partitionable implies $|C| < |C^1| \times |C^2|$.

In this subsection some basic concepts required to discuss the partitionability property will be presented. The underlying graph of a network is defined and its relationship to partitionability and other properties are discussed.

Definition 4.10:

Let $K \in K[V_I \times V_O]$, $K = <C>$. Let $G[V_I \times V_O] \triangleq \{<v_a,v_b> \in C_m \mid C_m \in C\}$. Then $G[V_I \times V_O]$ is the *underlying graph of K*.

Definition 4.11:

Let $G[V_I \times V_O]$ be the underlying graph of $K \in K[V_I \times V_O]$. Then the connected subgraphs of $G[V_I \times V_O]$ are called *graph components of* $G[V_I \times V_O]$.

Notation: Graph components are denoted by $G_1, G_2, \ldots, G_n$. Denote the vertices associated with G_r by $V_{I,r}$ and $V_{O,r}$, $V_{I,r} \subseteq V_I$, $V_{O,r} \subseteq V_O$. In a component G_r there exists a path from each node to every other node and there is no path between any two nodes from different components. Clearly $G[V_I \times V_O] = \bigcup_r G_r$, $\bigcup_r V_{I,r} = V_I$, and

$\bigcup_r V_{O,r} = V_O$.

Theorem 4.12:

Let $K \in K[V_I \times V_O]$, $K = <C>$. Let G be the underlying graph of K. K is a prime network iff G has a single component.

Definition 4.13:

Let $G[V_I \times V_O]$ be the underlying graph of $K \in K[V_I \times V_O]$, $K = <C>$. Let $C_m \in C$ and let G_r be a component of $G[V_I \times V_O]$. Define the *projection p of* C_m *onto* G_r as follows: $p(C_m,G_r) \triangleq \{<v_a,v_b> \in C_m \mid <v_a,v_b> \in G_r\}$.

Theorem 4.14:

Let $G[V_I \times V_O]$ be the underlying graph of $K \in K[V_I \times V_O]$, $K = <C>$. Let $C_m \in C$ and let $\{G_1,G_2,...,G_n\}$ be the set of all components of $G[V_I \times V_O]$. Then $C_m = p(C_m,G_1) \cup p(C_m,G_2) \cup \cdots p(C_m,G_n)$.

Definition 4.15:

Let $G[V_I \times V_O]$ be the underlying undirected graph of $K \in K[V_I \times V_O]$, $K = <C>$. Let G_i be a component of $G[V_I \times V_O]$. Define the *residue set modulo* G_i as follows: $r(G_i) \triangleq \{p(C_b,G_i) \mid \forall C_b \in C\}$.

Definition 4.16

Let G_r be a component of the underlying graph $G[V_I \times V_O]$ of $K \in K[V_I \times V_O]$, $K = <C>$. Let $r(G_r)$ be the residue set modulo G_r, G_r over $V_{I,r} \times V_{O,r}$, then $<r(G_r)> \in K[V_{I,r} \times V_{O,r}]$ is called a *component network of K* denoted by $K(G_r)$.

In this section the background on single stage networks was presented. This work was developed in [SeS85] and is presented here for completeness only (without proofs). Single stage network was defined and some basic properties of single stage partitionable networks were presented. The single stage network together with some other concepts

will be used to discuss multistage networks in the next section.

V Multistage Interconnection Networks

In this section of multistage partitionable networks will be studied. First a multistage network model is developed and then the multistage partitionable network is defined. Two types of partitionable networks are defined and two methodologies are developed each of which can be used to construct its class of partitionable networks. The material is presented for the case of partitionability of number of block networks of power of two however it can easily be generalized to powers of any integer.

Definition 5.1:

Let $C[V_I^1 \times V_O^1]$ and $C[V_I^2 \times V_O^2]$ be two C-sets. Define a *map* γ as follows:

$\gamma: C[V_I^1 \times V_O^1] \times C[V_I^2 \times V_O^2] \rightarrow C[V_I^1 \times V_O^2]$,

$C_a \gamma C_b \triangleq \{<u_i,v_l> \mid <u_i,v_j> \in C_a, <u_k,v_l> \in C_b, j=k\}$.

Definition 5.2:

Let $K^r \in K[V_I^r \times V_O^r]$ be a single stage network. Let $\omega^{r-1,r}$ be a nondestructive correspondence. $\omega^{r-1,r} \in C[V_O^{r-1} \times V_I^r]$, $s(\omega^{r-1,r}) = V_O^{r-1}$, $d(\omega^{r-1,r}) = V_I^r$.

Define $\circ$: $\omega^{r-1,r} \circ K^r = <\{\omega^{r-1,r} \gamma C_j^r \mid C_j^r \in C^r\}>$.

Note that $\omega^{r-1,r} \circ K^r \in K[V_I^{r-1} \times V_O^r]$.

Definition 5.3:

Let $\{K^r \mid r=0,1,...m-1\}$ be a set of single stage networks, $K^r \in K[V_I^r \times V_O^r]$.

Let $\{\omega^{r-1,r} \mid r=1,2,...m-1\}$ be a set of correspondences, $\omega^{r-1,r} \in C[V_O^{r-1} \times V_I^r]$, $s(\omega^{r-1,r}) = V_O^{r-1}$, and $d(\omega^{r-1,r}) = V_I^r$.

Define *multistage network* as follows: $K^0 \circ \omega^{0,1} \circ K^1 \circ \omega^{1,2} \circ \cdots K^{m-1}$.

Model similar to this was used by [Ben65] to study rearrangeable networks.

Notation:

$K^{i,j} = K^i \circ \omega^{i,i+1} \circ K^{i+1} \circ \omega^{i+1,i+2} \circ \cdots K^j$, $j \geq i+1$, $K^{i,j} = K^i$, $j=i$.

$C^{i,j} = C^i \gamma \omega^{i,i+1} \gamma C^{i+1} \gamma \omega^{i+1,i+2} \gamma \cdots C^j$, $j \geq i+1$, $C^{i,j} = C^i$, $j=i$.

Assume all K^r are reconfigurable. The case of K^r fixed network can be handled by absorbing it into the $\omega^{r-1,r}$ and $\omega^{r,r+1}$ for the purpose of the analysis. For an example of an arbitrary multistage interconnection network see Figure 2.

Definition 5.4:

Let $C_a \in C[V_I \times V_O]$. If

(1) $\forall u_a \in V_I \quad \exists! v_b \in V_O \ni <u_a,v_b> \in C_a$, (1:1), and

(2) $\forall v_c \in V_O \quad \exists u_d \in V_I \ni <u_d,v_c> \in C_a$, (onto),

then C_a is called *permutation correspondence.*

Note that this implies $|V_I| = |V_O|$ (finite sets) but not $V_I = V_O$.

Definition 5.5:

Let $K \in K[V_I \times V_O]$, $K = <C>$. If $\forall C_a \in C$, C_a is a permutation correspondence then K is called *permutation network.*

In the rest of this paper the discussion will restricted to permutation networks in particular the following assumptions are used hereafter.

(1) $|V_I^0| = N$.

(2) K^r, r=0,1,...m−1 is a permutation network.

(3) $\omega^{r-1,r}$, r=1,2,...m−1 is a permutation correspondence.

(4) $\log_2 N = m$.

(5) $m \geq 2$.

Note that (2) and (3) $\Rightarrow |V_I^r| = |V_O^r| = N$, r=0,1,...m−1.

Definition 5.6:

Let $K = K^{0,m-1}$ be a multistage network. *Fixing* K^r *in* C_x^r means selecting correspondence $C_x^r \in C^r$ and holding the K^r in that state.

Note that K^r fixed in $C_x^r \in C^r$ is equivalent to the fixed network $\hat{K}^r = <\{C_x^r\}>$. Note that K with K^r fixed is equivalent to $\hat{K} = K^{0,r-1} \circ \omega^{r-1,r} \circ <\{C_x^r\}> \circ \omega^{r,r+1} \circ K^{r+1,m-1}$. The K^j, $j \neq r$ networks are called *free networks* (stages).

The following development will discuss the class of σ-partitionable networks. It will be assumed that the number of "parts" is a power of two. This assumption is for the ease of presentation only and it should be clear that all results here are valid (with slight modifications) for number of "parts" of power of any integer base.

Definition 5.7:

Let K be a multistage network $K = K^{0,m-1}$. K is *simply partitionable* if fixing K^{m-1} will produce two (data path independent) networks $K_0^{0,m-2}$ and $K_1^{0,m-2}$ such that:

(1) $s(K_0^{0,m-2} \sigma K_1^{0,m-2}) = s(K^{0,m-2})$.

(2) $d(K_0^{0,m-2} \sigma K_1^{0,m-2}) = d(K^{0,m-2})$.

(3) $K_0^{0,m-2} \sigma K_1^{0,m-2} \supseteq c\ K^{0,m-2}$.

The networks $K_0^{0,m-2}$ and $K_1^{0,m-2}$ are called *block networks*. Note that for permutation networks (as in this discussion) (3) implies (1) and (2).

Definition 5.8:

Let K be a multistage network $K = K^{0,m-1}$. K is *simply partitionable isomorphic* if fixing K^{m-1} will produce two block networks $K_0^{0,m-2}$ and $K_1^{0,m-2}$ such that:

(1) K is simply partitionable with block networks $K_0^{0,m-2}$ and $K_1^{0,m-2}$.

(2) $K_0^{0,m-2} \approx K_1^{0,m-2}$.

Theorem 5.9:

Let $K = K^{0,m-1}$, $K \in K[V_I^0 \times V_O^{m-1}]$ be a multistage network. Let K be simply partitionable isomorphic with block networks $K_0^{0,m-2} \in K[V_{I,0}^0 \times V_{O,0}^{m-2}]$ and $K_1^{0,m-2} \in K[V_{I,1}^0 \times V_{O,1}^{m-2}]$.

Then $V_{I,0}^0 \cup V_{I,1}^0 = V_I^0$ and $V_{O,0}^{m-2} \cup V_{O,1}^{m-2} = V_O^{m-2}$.

Proof:

$K^{0,m-1}$ permutation network and $K_0^{0,m-2}$ σ $K_1^{0,m-2}$ $\supseteq c$ $K^{0,m-2}$ $\Rightarrow$ $V_{I,0}^0 \cup V_{I,1}^0 = V_I^0$ and $V_{O,0}^{m-2} \cup V_{O,1}^{m-2} = V_O^{m-2}$.

□

Theorem 5 10:

Let $K = K^{0,m-1}$, $K \in K[V_I^0 \times V_O^{m-1}]$ be a multistage network. Let K be simply partitionable isomorphic with block networks $K_0^{0,m-2} \in K[V_{I,0}^0 \times V_{O,0}^{m-2}]$ and $K_1^{0,m-2} \in K[V_{I,1}^0 \times V_{O,1}^{m-2}]$. Let K^{m-1} be fixed in C_x^{m-1}. Let $V_{I,j}^{m-1} \triangleq \{v_a \in V_I^{m-1} \mid v_b \in V_{O,j}^{m-2}, \ <v_b,v_a> \in \omega^{m-2,m-1}\}$ and $V_{O,j}^{m-1} \triangleq \{v_a \in V_O^{m-1} \mid v_b \in V_{I,j}^{m-1}, \ <v_b,v_a> \in C_x^{m-1}\}$, j=0,1.

Then $V_{I,0}^{m-1} \cup V_{I,1}^{m-1} = V_I^{m-1}$ and $V_{O,0}^{m-1} \cup V_{O,1}^{m-1} = V_O^{m-1}$.

Proof:

(1): Theorem 5.9 $\Rightarrow V_{O,0}^{m-2} \cap V_{O,1}^{m-2} = V_O^{m-2}$.

(2): (1) and $\omega^{m-2,m-1}$ permutation correspondence $\Rightarrow V_{I,0}^{m-1} \cup V_{I,1}^{m-1} = V_I^{m-1}$.

(3): (2) and C_x^{m-1} permutation correspondence $\Rightarrow V_{O,0}^{m-1} \cup V_{O,1}^{m-1} = V_O^{m-1}$.

□

The following is an algorithm to synthesize simply partitionable networks. Let $K = K^{0,m-1} = K^0 \circ \omega^{0,1} \circ K^1 \circ \omega^{1,2} \cdots K^{m-1}$, be a multistage network. Let $|V_I^0| = N$, $m = \log_2 N$. The construction of the partitionable network will be given in terms of constraints on K^r and the $\omega^{r,r+1}$ correspondences.

Algorithm 5.11:

Let $K = K^{0,m-1}$ be a multistage network. To construct a simply partitionable network the following constraints on K^r and on $\omega^{r,r+1}$ correspondences must be satisfied.

(1) For r=0,1,...m−2, K^r must have components K_0^r and K_1^r where $K_0^r \in K[V_{I,0}^r \times V_{O,0}^r]$ and $K_1^r \in K[V_{I,1}^r \times V_{O,1}^r]$.

(2) $K_0^r \approx K_1^r$.

(3) $d(K_0^{r-1}) \diamond \omega^{r-1,r} (V_{O,0}^{r-1} \times V_{I,0}^r) = s(K_0^r)$, r=m−1,m−2,...1. Where $\omega(V_{O,j} \times V_{I,j}) \triangleq \omega \bigcap (V_{O,j} \times V_{I,j})$ and $A \diamond \hat{\omega} \triangleq \{v_b \in d(\hat{\omega}) \mid \langle v_a, v_b \rangle \in \hat{\omega},\ v_a \in A\}$.

Proof:

(1): Algorithm constraint (1) ⇒ K^r has two components K_0^r and K_1^r, r=0,1,...m−2.

(2): (1) and Algorithm constraint (1) ⇒ $K^0 \circ \omega^{0,1} \circ K^1 \circ \cdots K^{m-2} \subseteq c$ $(K_0^0 \sigma K_1^0) \circ \omega^{0,1} \circ (K_0^1 \sigma K_1^1) \circ \omega^{1,2} \circ \cdots (K_0^{m-2} \sigma K_1^{m-2})$.

(3): (2) and Algorithm constraint (3) and property of ∘ distributing over σ ⇒ $K^0 \circ \omega^{0,1} \circ K^1 \circ \cdots K^{m-2} \subseteq c$ $(K_0^0 \sigma K_1^0) \circ \omega^{0,1} \circ (K_0^1 \sigma K_1^1) \circ \omega^{1,2} \circ \cdots (K_0^{m-2} \sigma K_1^{m-2}) = (K_0^0 \circ \omega^{0,1} (V_{O,0}^0 \times V_{I,0}^1) \circ K_0^1 \circ \omega^{1,2} (V_{O,j}^1 \times V_{I,0}^2) \circ \cdots K_0^{m-2}) \sigma (K_1^0 \circ \omega^{0,1} (V_{O,1}^0 \times V_{I,1}^1) \circ K_1^1 \circ \omega^{1,2} (V_{O,1}^1 \times V_{I,1}^2) \circ \cdots K_1^{m-2})$.

(4): Theorem 5.10 shows that since K^{m-1} is a permutation network any $C_x^{m-1} \in C^{m-1}$ can be used in fixing K^{m-1}.

(5): (3) and (4) $\Rightarrow$ $K^{0,m-1}$ is simply partitionable.

(6): Algorithm constraint (2) $\Rightarrow$ the components of K_0^r and K_1^r are isomorphic $\Rightarrow$ $K_0^r \approx K_1^r$.

(7): (5) and (6) $K^{0,m-1}$ is simply partitionable isomorphic network.

$\sqcap$

A note on the notation in this section. Let $K^r \in [V_I^r \times V_O^r]$ be a network, $V_I^r = \{v_0,v_1,\ldots v_{t-1}\}$ and $V_O^r = \{u_0,u_1,\ldots u_{t-1}\}$. Let $K_a^r \in [V_{I,a}^r \times V_{O,a}^r]$, be a component network $V_{I,a}^r = \{v_i,v_j,\ldots\}$ and $V_{O,a}^r = \{u_x,u_y,\ldots\}$. Since K^r is a permutation network therefore the component K_a^r is as well, and so it is always possible to relabel the vertices of V_I and V_O in such a way that the component network has $V_{I,a}^r = \{v_i,v_j,v_k,\ldots\}$ and $V_{O,a}^r = \{u_i,u_j,u_k,\ldots\}$. By proper encoding $V_{I,a}^r = \{p_0p_1 \cdots p_{(m-1)-r} \mid p_i=0,1\}$ and $V_{O,a}^r = \{q_0q_1 \cdots q_{(m-1)-r} \mid q_j=0,1\}$. Consequently K_a^r can be identified as $K^r_{p_0p_1 \cdots p_{(m-1)-r}}$.

Definition 5.12:

Let $K^{0,m-1} \in K[V_I^0 \times V_O^{m-1}]$, be a multistage network. If $K^{0,r}_{p_0p_1 \cdots p_{(m-1)-r}}$ is simply partitionable isomorphic with block networks $K^{0,r-1}_{p_0p_1 \cdots p_{(m-1)-(r+1)}0}$ and $K^{0,r-1}_{p_0p_1 \cdots p_{(m-1)-(r+1)}1}$, r=m−1,m−2,...1, then K is called *recursively partitionable isomorphic network.*

Theorem 5.13:

Let $K = K^{0,m-1}$, $K \in K[V_I^0 \times V_O^{m-1}]$ be a multistage network. Let $K_j^{0,r-1} \in K[V_{I,j}^0 \times V_{O,j}^{r-1}]$, be a simply partitionable isomorphic network with block networks $K_{j,0}^{0,r-2} \in K[V_{I,j,0}^0 \times V_{O,j,0}^{r-2}]$ and $K_{j,1}^{0,r-2} \in K[V_{I,j,1}^0 \times V_{O,j,1}^{r-2}]$, $j=0,1,\ldots 2^{(m-1)-(r-2)}-1$, r=m,m−1,...2.

Then $V^0_{I,j,0} \cup V^0_{I,j,1} = V^0_{I,j}$ and $V^{r-2}_{O,j,0} \cup V^{r-2}_{O,j,1} = V^{r-2}_{O,j}$.

Proof:

$K^{0,r-2}_{j,0}$ σ $K^{0,r-2}_{j,1}$ $\supseteq c$ $K^{0,r-2}_{j}$ and K is a permutation network $\Rightarrow$ $V^0_{I,j,0} \cup V^0_{I,j,1}$ $= V^0_{I,j}$ and $V^{r-2}_{O,j,0} \cup V^{r-2}_{O,j,1} = V^{r-2}_{O,j}$.

$\sqcap$

Theorem 5.14:

Let $K = K^{0,m-1}$, $K \in K[V^0_I \times V^{m-1}_O]$ be a multistage network. Let $K^{0,r-1}_j$ be simply partitionable isomorphic with block networks $K^{0,r-2}_{j,0} \in K[V^0_{I,j,0} \times V^{r-2}_{O,j,0}]$ and $K^{0,r-2}_{j,1} \in K[V^0_{I,j,1} \times V^{r-2}_{O,j,1}]$, with K^{r-1}_j fixed in C^{r-1}_x, $j=0,1,...2^{(m-1)-(r-2)}-1$, $r=m,m-1,...2$. Let $V^{r-1}_{I,j,k} \triangleq \{v_a \in V^{r-1}_{I,j} \mid v_b \in V^{r-2}_{O,j,k}, <v_b,v_a> \in \omega^{r-2,r-1}\}$ and $V^{r-1}_{O,j,k} \triangleq \{v_a \in V^{r-1}_{O,j} \mid v_b \in V^{r-1}_{I,j,k}, <v_b,v_a> \in C^{r-1}_x\}$, $k=0,1$.

Then $V^{r-1}_{I,j,0} \cup V^{r-1}_{I,j,1} = V^{r-1}_{I,j}$ and $V^{r-1}_{O,j,0} \cup V^{r-1}_{O,j,1} = V^{r-1}_{O,j}$.

Proof:

(1): Theorem 5.13 $\Rightarrow$ $V^{r-2}_{O,j,0} \cup V^{r-2}_{O,j,1} = V^{r-2}_{O,j}$.

(2): (1) and $\omega^{r-2,r-1}$ permutation correspondence $\Rightarrow$ $V^{r-1}_{I,j,0} \cup V^{r-1}_{I,j,1} = V^{r-1}_{I,j}$.

(3): (2) and C^{r-1}_x permutation correspondence $\Rightarrow$ $V^{r-1}_{O,j,0} \cup V^{r-1}_{O,j,1} = V^{r-1}_{O,j}$.

$\sqcap$

The notation of recursively partitionable networks will now change as follows. If $K^{0,r}_{p_0p_1 \cdots p_{(m-1)-r}}$ is a simply partitionable isomorphic network then the block networks are denoted as $K^{0,r-1}_{p_0p_1 \cdots p_{(m-1)-(r+1)}0}$ and $K^{0,r-1}_{p_0p_1 \cdots p_{(m-1)-(r+1)}1}$, $r=m-1,m-2,...0$. If $K^r_{p_0p_1 \cdots p_{(m-1)-r}}$, $r=m-1,m-2,...0$ is a nonprime network with two component networks then the component networks are denoted as $K^r_{p_0p_1 \cdots p_{(m-1)-(r+1)}0}$ and $K^r_{p_0p_1 \cdots p_{(m-1)-(r+1)}1}$.

Algorithm 5.15:

Let $K = K^{0,m-1}$ be a multistage network. To construct a recursively partitionable network $K = K^{0,m-1}$ the following constraints on K^r and on $\omega^{r,r+1}$ correspondences must be satisfied.

(1) For r=0,1,...m−2, $K^r_{p_0p_1\cdots p_{(m-1)-r}}$ must have components $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}0}$ and $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}1}$.

(2) $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}0} \approx K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}1}$.

(3) $\forall\, K^r_{p_0p_1\cdots p_{(m-1)-r}}$ $\exists!$ two component networks $K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}0}$ and $K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}$ $\ni$ $(d(K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}0}) \cup d(K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}))$ $\diamond\, \omega^{r-1,r}$ $(V^{r-1}_{O,p_0p_1\cdots p_{(m-1)-r}} \times V^r_{I,p_0p_1\cdots p_{(m-1)-r}}) = s(K^r_{p_0p_1\cdots p_{(m-1)-r}})$, r=m−1,m−2,...1.

Proof:

A. Show P_{m-1} holds. $P_{m-1} : K^{0,m-1}$ is simply partitionable isomorphic with blocks $K^{0,m-2}_0$ and $K^{0,m-2}_1$.

(1): Algorithm constraint (1) $\Rightarrow$ $K^r_{p_0p_1\cdots p_{(m-1)-r}}$ has two components $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}0}$ and $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}1}$, r=0,1,...m−2.

(2): Do the following process for r=0,1,...m−2 if $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}0}$ and $K^r_{p_0p_1\cdots p_{(m-1)-(r+1)}1}$ satisfy Algorithm (3) then form $K^r_{p_0p_1\cdots p_{(m-1)-r}}$ using the σ operation. Continue this process repeatedly until get components K^r_0 and K^r_1.

(3): (2) and Algorithm constraint (1) $\Rightarrow$ $K^0 \circ \omega^{0,1} \circ K^1 \circ \cdots K^{m-2} \subseteq c$ $(K^0_0 \,\sigma\, K^0_1) \circ \omega^{0,1} \circ (K^1_0 \,\sigma\, K^1_1) \circ \omega^{1,2} \circ \cdots (K^{m-2}_0 \,\sigma\, K^{m-2}_1)$.

(4): (3) and property of $\circ$ distributing over σ $\Rightarrow$ $K^0 \circ \omega^{0,1} \circ K^1 \circ \cdots K^{m-2}$ $\subseteq c$ $(K^0_0 \,\sigma\, K^0_1) \circ \omega^{0,1} \circ (K^1_0 \,\sigma\, K^1_1) \circ \omega^{1,2} \circ \cdots (K^{m-2}_0 \,\sigma\, K^{m-2}_1) =$

$(K_0^0 \circ \omega^{0,1}(V_{O,0}^0 \times V_{I,0}^1) \circ K_0^1 \circ \omega^{1,2}(V_{O,0}^1 \times V_{I,0}^2) \circ \cdots K_0^{m-2})\ \sigma\ (K_1^0$
$\circ \omega^{0,1}(V_{O,1}^0 \times V_{I,1}^1) \circ K_1^1 \circ \omega^{1,2}(V_{O,1}^1 \times V_{I,1}^2) \circ \cdots K_1^{m-2})$.

(5): Theorem 5.14 shows that since K^{m-1} is a permutation network any $C_x^{m-1} \in C^{m-1}$ can be used in fixing K^{m-1}.

(6): (4) and (5) $\Rightarrow$ $K^{0,m-1}$ is simply partitionable.

(7): Algorithm constraint (2) $\Rightarrow$ the components of K_0^r and K_1^r are isomorphic $\Rightarrow$ $K_0^r \approx K_1^r$.

(8): (6) and (7) $K^{0,m-1}$ is simply partitionable isomorphic network.

B. Show P_{r-1} assuming P_r holds. P_{r-1}: $K^{0,r-1}_{p_0p_1 \cdots p_{(m-1)-(r-1)}}$ is simply partitionable isomorphic with blocks $K^{0,r-2}_{p_0p_1 \cdots p_{(m-1)-(r)}0}$ and $K^{0,r-2}_{p_0p_1 \cdots p_{(m-1)-(r)}1}$, r=m,m−1,...2.

(1): Algorithm constraint (1) $\Rightarrow$ $K^k_{p_0p_1 \cdots p_{(m-1)-k}}$ has two components $K^k_{p_0p_1 \cdots p_{(m-1)-(k+1)}0}$ and $K^k_{p_0p_1 \cdots p_{(m-1)-(k+1)}1}$ for k=r−2,r−3,...0. r=m,m−1,...2.

(2): For k=r−2,r−3,...0 do the following process. If $K^k_{p_0p_1 \cdots p_{(m-1)-(k+1)}0}$ and $K^k_{p_0p_1 \cdots p_{(m-1)-(k+1)}1}$ satisfy Algorithm constraint (3) then combine them using σ to form $K^k_{p_0p_1 \cdots p_{(m-1)-k}}$. Continue repeatedly this process until $K^k_{p_0p_1 \cdots p_{(m-1)-(r-1)}}$ is formed.

(3): (2) and Algorithm constraint (1) $\Rightarrow$ $K^0_{p_0p_1 \cdots p_{(m-1)-(r-1)}} \circ$
$\omega^{0,1}(V^0_{O,p_0p_1 \cdots p_{(m-1)-(r-1)} \times V^1_{I,p_0p_1 \cdots p_{(m-1)-(r-1)}}}) \circ K^1_{p_0p_1 \cdots p_{(m-1)-(r-1)}} \circ \cdots$
$K^{r-2}_{p_0p_1 \cdots p_{(m-1)-(r-1)}} \subseteq_c (K^0_{p_0p_1 \cdots p_{(m-1)-(r)}0}\ \sigma\ K^0_{p_0p_1 \cdots p_{(m-1)-(r)}1}) \circ$
$\omega^{0,1}(V^0_{O,p_0p_1 \cdots p_{(m-1)-(r-1)} \times V^1_{I,p_0p_1 \cdots p_{(m-1)-(r-1)}}}) \circ$
$(K^1_{p_0p_1 \cdots p_{(m-1)-(r)}0}\ \sigma\ K^1_{p_0p_1 \cdots p_{(m-1)-(r)}1}) \circ \cdots$
$(K^{r-2}_{p_0p_1 \cdots p_{(m-1)-(r)}0}\ \sigma\ K^{r-2}_{p_0p_1 \cdots p_{(m-1)-(r)}1}) = (K^0_{p_0p_1 \cdots p_{(m-1)-(r)}0} \circ$
$\omega^{0,1}(V^0_{O,p_0p_1 \cdots p_{(m-1)-(r)}0} \times V^1_{I,p_0p_1 \cdots p_{(m-1)-(r)}0}) \circ K^1_{p_0p_1 \cdots p_{(m-1)-(r)}0} \circ \cdots$

$$K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r)}0}) \;\sigma\; (K^{0}_{p_0p_1\cdots p_{(m-1)-(r)}1} \circ$$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r)}1} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r)}1}) \circ K^{1}_{p_0p_1\cdots p_{(m-1)-(r)}1} \circ \cdots$$

$$K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r)}1}).$$

(3.1): P_r holds $\Rightarrow K^{0}_{p_0p_1\cdots p_{(m-1)-r}} \circ$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-r}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-r}}) \circ K^{1}_{p_0p_1\cdots p_{(m-1)-r}} \circ \cdots$$

$$K^{r-1}_{p_0p_1\cdots p_{(m-1)-r}} \subseteq c\; (K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)}0} \;\sigma\; K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}) \circ$$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-r}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-r}}) \circ \cdots$$

$$(K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}0} \;\sigma\; K^{r-1}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}).$$

(3.2): (3.1) $\Rightarrow K^{0}_{p_0p_1\cdots p_{(m-1)-r}} \circ \omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-r}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-r}})$

$$\circ K^{1}_{p_0p_1\cdots p_{(m-1)-r}} \circ \cdots K^{r-2}_{p_0p_1\cdots p_{(m-1)-r}} \subseteq c$$

$$(K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)}0} \;\sigma\; K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}) \circ$$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r+1)}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r+1)}}) \circ \cdots$$

$$(K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r+1)}0} \;\sigma\; K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r+1)}1}).$$

(3.3): Substituting (3) into (3.2) $\Rightarrow K^{0}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \circ$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r-1)}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r-1)}}) \circ K^{1}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \circ \cdots$$

$$K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \subseteq c \underset{ij=00}{\overset{11}{\sigma}} (K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}) \circ$$

$$\bigcup_{ij=00}^{11} \omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}) \circ$$

$$\underset{ij=00}{\overset{11}{\sigma}} (K^{1}_{p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}) \circ \cdots \underset{ij=00}{\overset{11}{\sigma}} (K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}).$$

(3.4): (3.3) and property of $\circ$ distributing over $\sigma \Rightarrow K^{0}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \circ$

$$\omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r-1)}} \times V^{1}_{p_0p_1\cdots p_{(m-1)-(r-1)}}) \circ K^{1}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \circ \cdots$$

$$K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r-1)}} \subseteq c \underset{ij=00}{\overset{11}{\sigma}} (K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}) \circ$$

$$\bigcup_{ij=00}^{11} \omega^{0,1}\,(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r+1)^{ij}}}) \circ$$

$$\overset{11}{\underset{ij=00}{\sigma}}(K^{1}_{p_0p_1\cdots p_{(m-1)-(r+1)}ij}) \circ \cdots \overset{11}{\underset{ij=00}{\sigma}}(K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r+1)}ij}) =$$

$$\overset{11}{\underset{ij=00}{\sigma}}(K^{0}_{p_0p_1\cdots p_{(m-1)-(r+1)}ij} \circ \omega^{0,1}(V^{0}_{O,p_0p_1\cdots p_{(m-1)-(r+1)}ij} \times V^{1}_{I,p_0p_1\cdots p_{(m-1)-(r+1)}ij}) \circ$$

$$(K^{1}_{p_0p_1\cdots p_{(m-1)-(r+1)}ij}) \circ \cdots K^{r-2}_{p_0p_1\cdots p_{(m-1)-(r+1)}ij}).$$

(5): The rest is similar to the proof of P_{m-1} steps (5)-(8).

□

In this section the multistage networks were discussed. The multistage network model was developed and then two types of multistage partitionable networks were defined. Two methods were developed each synthesizing a class of multistage partitionable network.

VI Conclusions

Two methods are used to speed up the execution of computational task. One is new technology insertion and the other is the exploitation of parallelism in the computation. To take an advantage of the parallelism in a task requires the utilization of parallel computer architectures. At a certain high level of abstraction a reconfigurable parallel computer system is represented as a graph structure where the node represent processors, memories, or other devices and the labeled edges represent the communication states of the network.

In this research a topological property of interconnection networks partitionability was studied. The uses and advantages of partitionable networks are presented in a number of publications and summarized in the introduction. In particular a general graph/algebraic model of multistage network was developed. Two types of partitionable multistage networks were formally defined. Two methods were developed,

one for the synthesis of simply partitionable networks and another for the synthesis of recursively partitionable multistage networks. The algorithms are presented for the case where the number of block networks is a power of two, however they can easily be generalized to the power of any integer.

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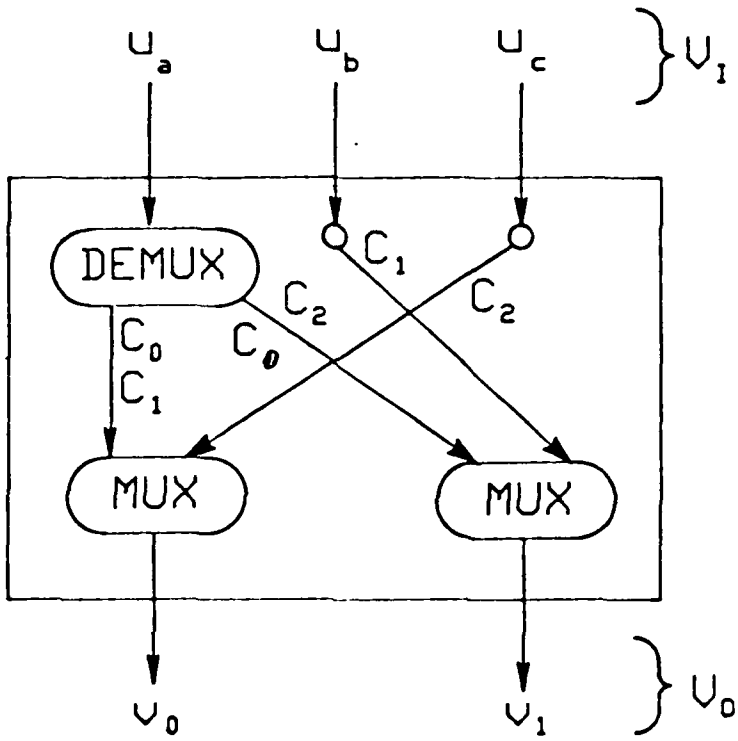

Figure 1:
An example of an arbitrary single stage network.

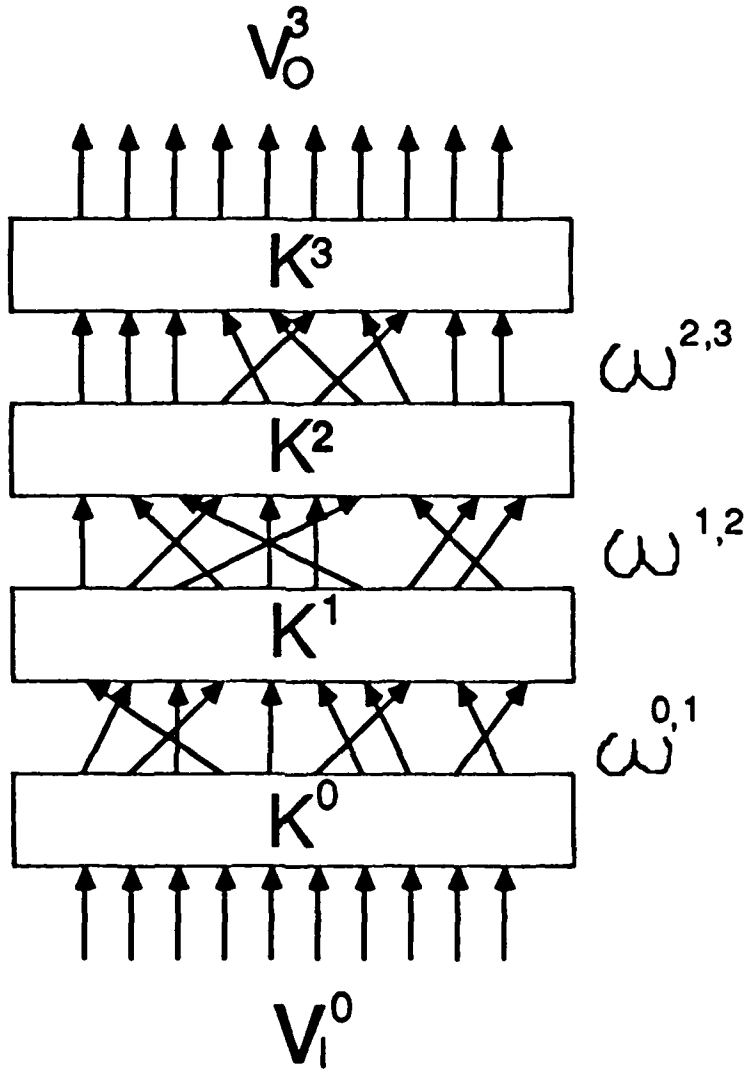

Figure 2:
An example of an arbitrary multistage network.

END

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